

High accuracy speed measurement using GPS (Global Positioning System)

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Abstract.

This article demonstrates that speed measurement with accuracy approaching 0.01 knot is possible by using GPS Doppler data. The method is illustrated with measurements made by GT-11 GPS unit made by Locosys.

Introduction

Typical approach to using GPS for speed measurement today is to consider a series of “trackpoints” that record position estimates (latitude and longitude) determined by the GPS at regular time intervals.

Each GPS trackpoint is determined with some error that is variable and difficult to determine. Hence, speed values computed from a series of trackpoints have unknown accuracy and cannot be considered reliable. It is virtually *impossible* to prove the accuracy of speed computed from a recorded series of trackpoints.

The most inaccurate is the method that tries to estimate an average speed over some “accumulated distance” between trackpoints. Due to trackpoint inaccuracies, the line connecting all track points is a zig-zag, even if the real path of a speed competitor is a smooth or straight line. Since the length of this zig-zag is always longer than a smooth/straight line, the “average speed” determined with the “accumulated distance” method *always overestimates* the real speed. The less accurate are trackpoints (the less accurate is a GPS unit) – the larger the estimated “average” speed and the more impressive is the “achievement”...

Doppler

An alternative to measuring speed from series of trackpoints is using the Doppler effect.

Modern GPS devices implement digital PLL (phase-lock loop) receivers to continuously track carrier frequencies of a number of satellites. For example, GT-11 tracks carrier frequencies of up to 12 satellites simultaneously. The frequency tracking *has to* be continuous simply because each receiver has to be *always ready* to receive data from its satellite.

The very fact that data is read from any given satellite is proof that its carrier frequency is tracked with high accuracy.

The difference between the known satellite carrier frequency and the frequency determined at the receiver is known as a “Doppler shift”. This Doppler shift is *directly proportional* to velocity of the receiver along the direction to the satellite, regardless of the distance to this satellite.

With multiple satellites tracked it is possible to determine the 3D velocity vector of the receiver. In general, the more satellites are tracked – the better the speed estimate.

Accuracy of the Doppler tracking

The Doppler speed measurement accuracy is not constant. It depends on the number of tracked satellites as well as on their geometrical distribution above the horizon.

For this reason it is important to measure this accuracy directly - together with the actual speed determined from the Doppler shift if possible.

The most convenient way of verifying the Doppler speed measurement accuracy is recording the Doppler speed data of a stationary GPS receiver at regular time intervals. This method accounts for all known⁶ GPS-Doppler speed measurement errors. Results of @1 hour recording using GT-11 are presented in Fig.1.

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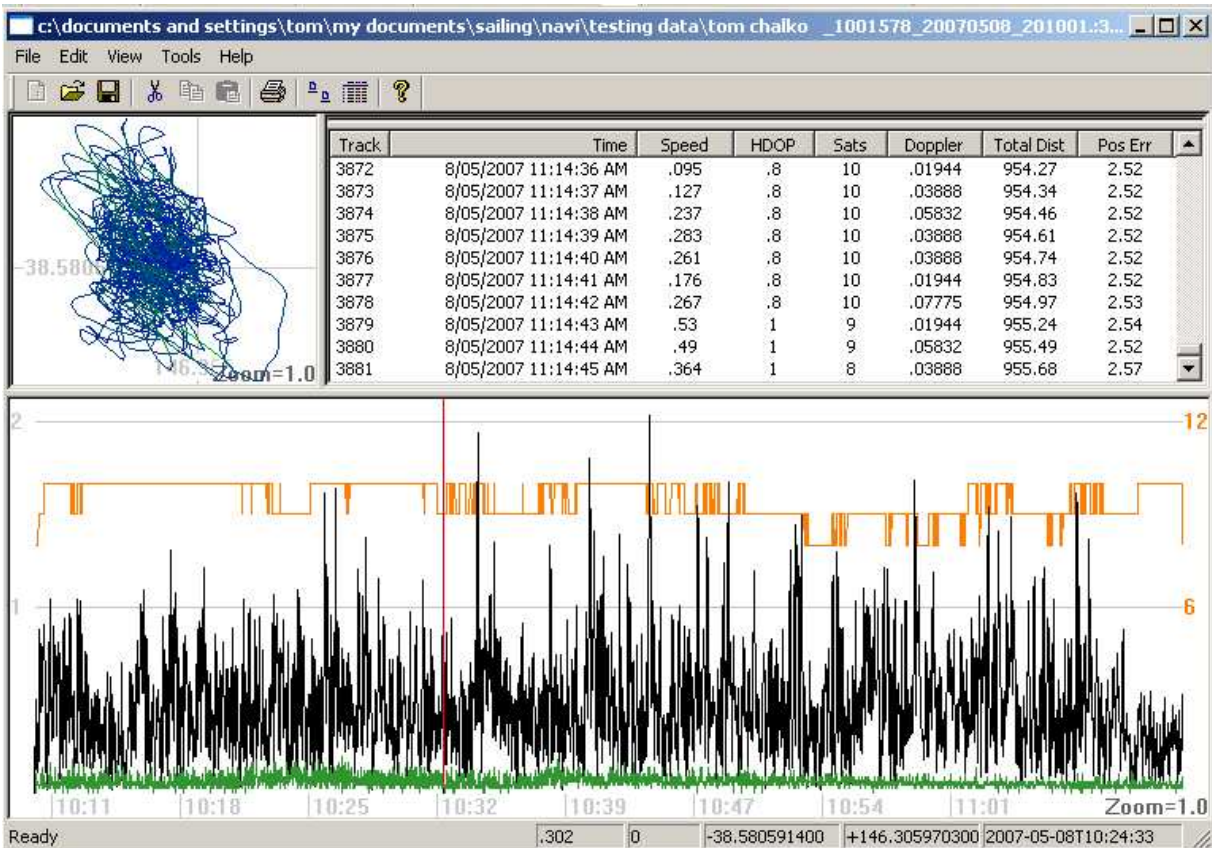


Fig.1. Comparison of **Doppler speed** and **trackpoint-derived speed** for stationary GT-11 using RealSpeed software. Binary GT-11 data was logged for about 1 hour every second. **Top-left:** latitude-longitude trackpoint trajectory. **Top-right:** a fragment of the data file (distance in meters, speed in knots). **Bottom:** speed in knots: **black** – speed determined from trackpoints, **green:** Doppler speed, **orange:** number of tracked satellites. Horizontal axis shows time in hours:minutes. The measured average Doppler speed is 0.0554 knot. The position-derived average speed is 0.479 knots. The GPS unit was stationary, but the accumulated distance computed from positions (trackpoints) is almost 1 km. Note the difference between HDOP (Horizontal Dilution of Precision) and the gps-chip computed Position Error for each trackpoint.

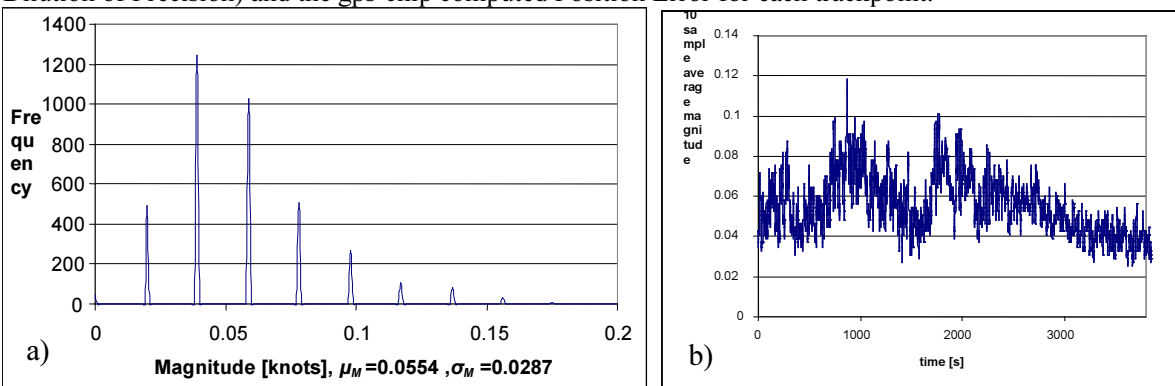


Fig.2. **a)** Measured distribution of the magnitude of Doppler measurement error in 1-second samples during 1-hour observation. Discrete resolution is clearly visible with 1-bit corresponding to about 0.0194 knot (0.01 m/s). **b)** Doppler measurement error as a function of time. 10-sample average error magnitude is shown. 1-bit resolution is constant (a), but the Doppler measurement accuracy varies in time.

Doppler accuracy at non-zero speed

What will happen to the Doppler speed resolution and accuracy for non-stationary GPS?

We need to remember that Earth spins. A point that seems stationary with respect to the surface of the Earth actually moves West-to-East with speed in the order of 700 knots in the geocentric frame of reference. (698 knots

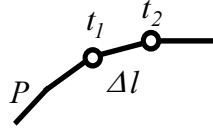
at Sandy Point, Vic, Australia). Yes, every parked car can receive a speeding fine !!!

Hence, from the point of view of the PLL Doppler frequency tracking system implemented in GPS devices it actually does not matter whether we try to measure speed of 700 knots, 650 knots or 750 knots.

Improving the accuracy by averaging

In their wisdom, developers of GT-11 enabled continuous logging of low-pass filtered Doppler speed data at 1 second intervals. This enables us to evaluate the average speed with greater accuracy than the accuracy of a single Doppler sample.

Let's begin with recalling the universally accepted definitions of speed and the average speed. Consider object moving along path P and traveling the distance Δl during the time interval $\Delta t = t_2 - t_1$.



The speed v of the object is defined as

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta l}{\Delta t} = \frac{dl}{dt} \quad (1)$$

The average speed \hat{v} over time T is defined as a time average of v as follows:

$$\hat{v} = \frac{\int_{t=0}^T v dt}{T} = \frac{\int_0^L dl}{T} = \frac{L}{T} \quad (2)$$

Definition of the average speed determines unique relationship between the travelled distance L (measured along the path P) and the average speed \hat{v}

$$L = \hat{v} T \quad (3)$$

It is important to point out that the average speed over the time T and the average speed over the distance L are identical: *there exists only one average speed that describes motion*. For this reason, the average speed is the best possible measure of speed in sports that use speed for their ranking.

In real measurements, each discrete GPS-Doppler speed sample v_{kD} contains measurement error v_{ke} . Hence true speed v_k is

$v_k = v_{kD} - v_{ke}$. For a discrete series of speed samples v_{kD} acquired at N uniform time intervals over the time T we can approximate the integral in the equation (2) by a sum and the expression for the unknown average speed becomes

$$\hat{v} = \frac{\sum_{k=1}^N v_{kD}}{N} - \frac{\sum_{k=1}^N v_{ke}}{N} = \hat{v}_D - \hat{v}_e \quad (4)$$

The first term \hat{v}_D is the exact average of all measured Doppler samples; the second term \hat{v}_e represents the measurement error. Other methods of integration of the discrete series are discussed later in this article.

Measurement errors v_{ke} of each sample can be considered a series of independent random variables that share the same probability distribution D . Fig 2 demonstrates that both the expected value μ and the standard deviation σ of D exist and are finite. A typical measured distribution of the magnitude of the speed error v_{ke} is shown in Fig 2. In reality, the probabilities of errors v_{ke} increasing and decreasing the measured values of speed are equal. Hence each random variable v_{ke} can be considered to have $\mu = 0$ and $\sigma < \sigma_M + \mu_M$, where μ_M is the expected value and σ_M is the standard deviation of the measured magnitude of the speed error v_{ke} .

The standard error $\sigma_{\hat{v}}$ of the measured average speed \hat{v} can be determined directly from the Central Limit Theorem⁵ to be:

$$\sigma_{\hat{v}} = \sigma_{\hat{v}_e} = \frac{\sigma}{\sqrt{N}} < \frac{\mu_M + \sigma_M}{\sqrt{N}} \quad (5)$$

This error is inversely proportional to \sqrt{N} and hence it decreases with the number of sampling intervals. As the number of sampling intervals N increases, the distribution of $\sigma_{\hat{v}}$ approaches the normal distribution, enabling us to determine the claimed average speed \hat{v} with any required confidence level.

In the absence of measured μ_M and σ_M the standard error σ specified by the GPS manufacturer for Doppler speed should be used.

Non-uniform sampling intervals

When multiple GT-11 units are used to sample GPS-Doppler speed at 1Hz each, it is possible to synchronize their sampling sequences to achieve GPS-Doppler speed samples at sub-second intervals, because all GT-11 units report their samples in UTC time with 1ms accuracy. Initially set time “skewing” has been found maintained for many hours (providing that all synchronized GT-11 units kept tracking satellites) and hence provides us with a method to increase the GPS-Doppler sampling frequency.

Ideally, when K GT-11 units are used, each second should be divided into K equal sampling intervals. However, in practice, K sub-second intervals may not be equal. For this reason we need to determine the average speed and its standard error for such a situation.

Without loss of generality we may consider the case of $K=2$ GT-11 units and the corresponding 2 sub-second time intervals Δt_1 and Δt_2 , because as we shall soon see the $K=2$ case is extendable to any K . The integral in equation (2) can be approximated by a discrete sum of samples as follows

$$\hat{v} = \frac{\sum_{k=1}^{N_1} v_{kD} \Delta t_1 + \sum_{k=1}^{N_2} v_{kD} \Delta t_2}{N_1 \Delta t_1 + N_2 \Delta t_2} + \frac{\sum_{k=1}^{N_1} v_{ke} \Delta t_1 + \sum_{k=1}^{N_2} v_{ke} \Delta t_2}{N_1 \Delta t_1 + N_2 \Delta t_2} = \hat{v}_D - \hat{v}_e \quad (6)$$

where N_1 and N_2 are numbers of sampling intervals Δt_1 and Δt_2 respectively in T and $N = N_1 + N_2$. The first term represents \hat{v}_D (the exact average of all measured Doppler samples); the second term represents \hat{v}_e (the measurement error). Applying the Central Limit Theorem⁵ to each term in the numerator of \hat{v}_e and implementing the theorem governing addition of variances of independent variables⁸ we obtain the following expression for the standard error $\sigma_{\hat{v}}$ of the average speed \hat{v} :

$$\sigma_{\hat{v}} = \sigma_{\hat{v}_e} = \sigma \frac{\sqrt{N_1 \Delta t_1^2 + N_2 \Delta t_2^2}}{N_1 \Delta t_1 + N_2 \Delta t_2} \quad (7)$$

This formula is easily extendable to accommodate K different sampling intervals as follows:

$$\sigma_{\hat{v}} = \sigma_{\hat{v}_e} = \sigma \frac{\sqrt{\sum_{k=1}^K N_k \Delta t_k^2}}{\sum_{k=1}^K N_k \Delta t_k} \quad (8)$$

For $K=1$ this formula gives equation (5) with N being the number of sampling intervals Δt during the time T .

Rectangular rule of integration

So far we approximated the integral in the equation (2) by a simple sum of discrete terms. This method of integration is referred to as a “rectangular rule of integration”⁸. In this rule it is assumed that the integrated function does not change between samples.

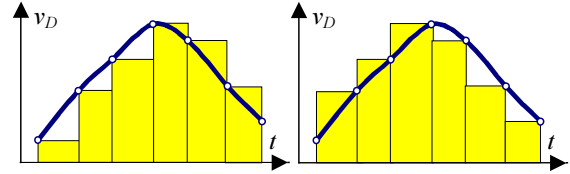


Fig 3. Comparison of Left Riemann and Right Riemann approximation in the rectangular rule of integration.

For a large number of discrete samples Right Riemann, Left Riemann approximations (as well as midpoint approximation that is not a convenient option for samples of an unknown function) methods converge to the same result, but it is clear that for a limited number of experimentally acquired samples the rectangular rule of integration can be a source of ambiguity.

Trapezoidal Rule of integration

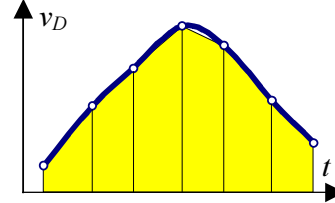


Fig 4. Trapezoidal rule of integration offers excellent accuracy for a limited number of discrete samples and eliminates the ambiguity associated with the rectangular rule of integration

Integration accuracy of an unknown function represented by a limited number of discrete samples can be improved and ambiguity associated with the rectangular rule of integration can be eliminated by implementing the so-called “trapezoidal rule of integration”⁸.

It is important to point out that in the rectangular method of integration the number of GPS-Doppler samples N is equal to the number of the sampling intervals during the time T (see Fig 3) and all samples are given the same weight.

In contrast, *the trapezoidal rule requires one extra sample and requires the first and the last sample in the interval T to be given weight of 0.5.*

For a uniform sampling period ($K=1$) the trapezoidal rule can be expressed as follows:

$$\hat{v}_D = \frac{\sum_{k=1}^N v_{kD}}{N} - \frac{v_{0D} + v_{ND}}{2N} \quad (9)$$

where v_{0D} and v_{ND} are the first and the last samples in the integration interval T . The expression for \hat{v}_D in the general case of K durations of sampling intervals is:

$$\hat{v}_D = \frac{\sum_{m=1}^K \sum_{k=1}^{N_k} v_{kD} \Delta t_m}{\sum_{k=1}^K N_k \Delta t_k} - \frac{v_{0D} \Delta t_1 + v_{ND} \Delta t_K}{2 \sum_{k=1}^K N_k \Delta t_k} \quad (10)$$

where $N = \sum_{k=1}^K N_k$, and Δt_1 and Δt_K are the corresponding (the first and the last) sampling intervals.

Using the same tools as in the previous section of this article we can estimate the standard error $\sigma_{\hat{v}}$ of the unknown average speed \hat{v} obtained using the trapezoidal rule of integration to be:

$$\sigma_{\hat{v}} = \sigma \sqrt{\frac{\sum_{k=1}^K N_k \Delta t_k^2 - \frac{\Delta t_1^2}{2} - \frac{\Delta t_K^2}{2}}{\sum_{k=1}^K N_k \Delta t_k}} \quad (11)$$

For $K=1$ this formula gives the equation (5) providing that N_k is the number of sampling intervals Δt_k during the time T .

Simpson Rule of integration

The so-called Simpson rule⁸ of integration of discrete series of samples uses parabolas to interpolate between discrete samples and is regarded as more accurate than the trapezoidal rule. Although it is possible to use the Simpson rule to compute \hat{v}_D and estimate the standard error $\sigma_{\hat{v}}$ of the corresponding average speed, the gain in the accuracy is likely to be too small to be worth an effort.

Average of M speed attempts

In some sports ranking is based on the average speed of several attempts (runs). If the number of attempts is M , the standard error of their average speed is:

$$\sigma_{\hat{v}_e} = \frac{1}{M} \sqrt{\sum_{k=1}^M \sigma_k^2}, \quad (12)$$

where σ_k is the standard error of the average speed of the k -th attempt.

Claimed speed

In sports that base their rankings on the achieved average speed it is convenient to introduce the concept of “claimed average speed”, being the average speed that can be claimed to have been achieved with the required level of confidence in the presence of measuring errors. Let’s define the claimed speed as

$$\hat{v}_{CLAIMED} = \hat{v}_D - c \sigma_{\hat{v}} \quad (13)$$

Fig 5 illustrates the fact that by adopting $c=2$ we can claim with 97.725% confidence that the claimed average speed has been achieved.

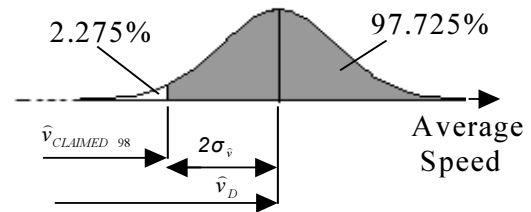


Fig. 5. Relationship between the measured average speed \hat{v}_D and claimed speed for $c=2$

The graph in Fig 6. illustrates relationship between the coefficient c and the corresponding confidence level of the claimed speed.

It is clear that if we ignore measurement errors ($c=0$) we can claim with 50% confidence that \hat{v}_D has been achieved.

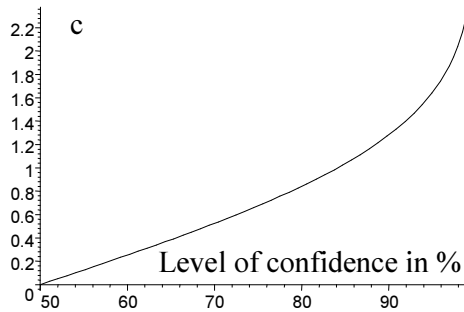


Fig 6. Relationship between the coefficient c and the confidence level of the claimed speed in the range 50% to 99% for normal distribution of σ_v

Increasing N

Regardless of the method of integration used, the accuracy of the average speed increases with the total number of samples and sampling intervals N in the interval T . N can be increased in three ways: increasing the time T for measuring the speed and/or increasing the sampling frequency and/or increasing the number of GT-11 units that measure the same average speed.

Increasing the number of GT-11 units to measure the same average speed (using an array of GPS units) has rather profound consequences for the accuracy of speed measurement using the GPS-Doppler method.

Theoretically speaking, very high accuracy in the average speed measurement can be achieved using the GPS-Doppler method, providing that sufficiently large GPS instrument array is used.

This is possible, because

1. The measurement error decreases with increased number of samples N
2. GPS-Doppler speed samples acquired by individual GPS units are independent

Examples

If a speed contest was made at the location and during the time I recorded the data presented in Fig 1. and Fig 2, an average of 10 consecutive 1-second Doppler readings could produce an average speed over 10 seconds with accuracy of 0.044 knot and 95% confidence (assuming $\sigma = 0.0841$ knot). The accuracy of the average speed of 5 * 10-second intervals would be 0.019 knots.

If **four** GT-11 units were used simultaneously for a 20 second run (500m run at ~50 knots), the speed measurement accuracy should be about 0.015 knot with 95% confidence and 0.019 knot with 98% confidence.

Proof of Speed

Perhaps the most important feature of the GPS-based Doppler method for measuring speed is that it can actually provide a proof of speed.

Using the Doppler data it is possible to prove that certain average speed with respect to the ground was maintained for a certain amount of time, because the Doppler method is trackable back to Units of Measurement.

Proof of speed is possible even though we may not be able to prove an exact location where that speed was achieved.

Incidentally, this is not the first time such a problem occurred. It is well known in physics that it is impossible to determine both velocity and position of an electron. We can accurately determine one or the other, but not both...

Form my tests presented in Fig 1 it is obvious that GPS trackpoints should never be used to estimate the speed. Trackpoints can only serve as a verification of the integrity of the Doppler speed data.

Setting up a Speed Contest

Since the Doppler speed accuracy varies with the number of tracked satellites and their distribution above the horizon, there is a need to determine and *monitor* this accuracy during every serious speed event.

The variability range of the standard speed error σ is not large, but observable. In two days of experimenting near Sandy Point, Victoria, Australia I observed the average Doppler standard speed error σ values ranging from 0.05 to 0.1 knots. Fig 1 (bottom) and Fig 2b illustrate this variability.

One way to determine the actual accuracy of Doppler speed measurement during any given event is to use 2 identical GPS units like GT-11 for the entire duration of the event, one unit placed in a stationary location and the other placed on a moving craft or person. The theoretical basis for this procedure is the fact that reasons for Doppler signal errors⁶ are common for multiple units that are sufficiently close to one another and hence the corresponding speed error distributions are correlated.

Care should be taken that a similar number of satellites are visible from both units. In case of speed windsurfing for example this is equivalent to the requirement of installing the GPS unit on top of a helmet so that is never under water.

An alternative method would be to use one GPS unit, but make it motionless for a few minutes before and after each speed attempt.

Doppler used in track data

It is important to note, that 0.05 m/s (@0.1 knot) accuracy in speed over 1 second interval is equivalent to 5 cm track position accuracy.

GPS chipset developers know this very well and actually use Doppler speed measurement to improve the accuracy of trackpoints.

Unfortunately, algorithms that mix Doppler and satellite distance data are proprietary and their unknown properties cannot be used to prove the speed or position.

Aliasing

The accuracy analysis presented above in this article is valid only when GPS-Doppler speed samples are free from aliasing errors.

Aliasing may occur if the bandwidth of the sampled Doppler process is too high in comparison to the sampling frequency. The Nyquist criterion requires the sampling frequency to be at least twice the maximum frequency present in the sampled process.

When a satellite phase is steady enough to admit this satellite data for position and speed calculations, the bandwidth of a typical Phase Lock Loop filter that tracks this satellite signal is 2Hz.

This means that in order to eliminate aliasing errors the following solutions are available:

1. Use 4Hz GPS-Doppler sampling rate. This can be accomplished either by using one GPS unit that is capable of reporting Doppler samples at 4Hz or four 1Hz GPS units synchronized to provide Doppler samples approximately every 250ms.
2. When sampling frequency is limited to F (for example 2Hz for twin GT-11 units), mechanical motion of the GPS unit used to measure speed should not contain frequencies above $F/2$ (1Hz in our example). Mechanical vibrations of the GPS unit with frequencies above $F/2$ (above 1Hz in our example) should be filtered either using a seismic suspension or biofeedback (locating the GPS on a head of the competitor).

Implementation of the second solution is described in the Reference 7.

Spoofing

Opponents of direct measurements of average speed claim that it is possible to boost the average speed result by inducing transverse oscillations and thereby increasing the length L of the travelled trajectory. Let's explore the upper limit of this boost in the sport of speed windsurfing.

Consider transverse harmonic oscillations with peak-to-peak stroke S [m], and frequency f [Hz] superimposed on motion with velocity V [knots]. When speed is sampled with frequency F , the maximum frequency of oscillations that can be recorded is $f=F/2$.

If oscillations are optimally phase-synchronized with the sampling process, samples will contain the maximum possible average transverse velocity SF during the stroke. The corresponding speed boost ε_B is:

$$\varepsilon_B = \sqrt{V^2 + (1.944SF)^2} - V \text{ [knots]},$$

where the factor 1.944 converts m/s to knots. For sampling rate $F=1$ Hz and speed $V=40$ knots this means that a machinery of 1m in size that generates motion precisely phase synchronized with the GPS sampling process is required to boost the average speed by just 0.05 knots. Such machinery is not only illegal in sailing, but will introduce extra drag and hence reduce the sailing speed.

For many reasons (such as satellite visibility, signal strength, reliability and accuracy of GPS measurements) it is recommended that speed-recording GPS units to be worn on/inside a helmet⁷ worn by a sailor. The stroke S of human head is limited to about 0.1 m. The corresponding upper limit of speed boost that can be achieved at $V=40$ knots is 0.00047 knots.

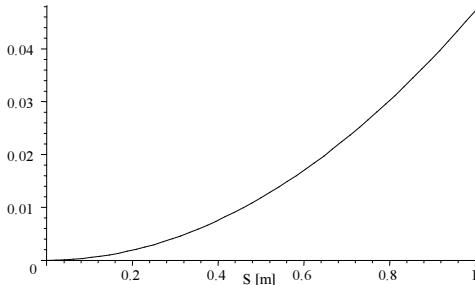


Fig 7. Maximum possible speed boost [knots] as a function of stroke for 1Hz speed samples at $V=40$ knots.

At World Record speed $V=50$ this upper limit for the speed boost is 0.00038 knots.

Let's now explore increasing the frequency of oscillations f to increase the speed boost. Human head cannot move with frequency faster than 2 Hz and stroke larger than about 5cm (0.05m). Due to the upper frequency limit, the maximum possible speed boost would be achieved when speed is sampled at 4 Hz. The corresponding "maximum physiologically achievable speed boost" at $V=50$ knots would be 0.0015 knots. Speed sampling at rates higher than 4 Hz will

actually reduce the average speed boost, because not all samples used in the average speed calculation will contain the full amount of the boosted speed.

In view of the above analysis we have to conclude that, providing that aliasing is eliminated, effects of transverse oscillations can be considered insignificant for speeds above 30 knots when speed recording GPS units are installed on/inside helmets of windsurfing sailors.

Future of GPS technology

Today GT-11 unit provides 0.01 m/s speed measurement resolution and 0.1 m/s accuracy for each one-second sample.

The short term future of hand-held GPS devices can be predicted by studying specifications of newly released GPS microprocessor chipsets that haven't yet found their way to the mass market.

These specifications indicate that as soon as 2008 we can expect GPS devices offering 0.01 m/s Doppler speed accuracy for each sample. It is rather interesting that the positional (trackpoint) accuracy will remain around 2.5 meters.

This seems yet another significant argument to commit to Doppler method for speed measurement.

Conclusions

- Doppler shift is directly proportional to speed. Hence, measuring Doppler shift is the most direct and hence the most accurate way of measuring speed.
- Doppler frequency is relatively insensitive to distances from satellites, phase delays and many other factors⁶ that are major sources of errors for trackpoints
- Doppler method of speed measurement provides proof of speed, because it is trackable to Units of Measurement
- Tolerance and accuracy of Doppler measurement can easily be measured and monitored experimentally

- Accuracy of Doppler speed measurements can be significantly improved by adopting **the average speed** as a measure of speed
- Accuracy of the average speed measurement over given amount of time interval increases with the number of Doppler speed samples in this interval.
- **Very high accuracy** in the average speed measurement can be achieved today providing that sufficiently large number of suitable GPS instruments (GT-11) is used to measure the speed
- Doppler speed measurements are repeatable and reproducible with experimentally verifiable accuracy and resolution
- The most practical method of computing the average speed from GPS-Doppler samples is to use the trapezoidal rule of integration.

GPS Doppler tracking data from multiple satellites provides a very accurate and very easy way of measuring average speeds. The longer the measuring period is, the more frequent are Doppler speed samples and the more GPS units are used simultaneously to measure the same speed - the better the accuracy.

For a speed event of 20 seconds or more, the average speed measurements with accuracy as high as 0.01 knot is possible with technology that exists **today**: the GT-11 unit from Locosys.

Are we ready to know the Real Speed we achieve?

Really?

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