Estimating Accuracy of GPS Doppler Speed Measurement using Speed Dilution of Precision (SDOP) parameter

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Abstract

This article presents a method for estimating the accuracy of GPS-Doppler speed measurement using Speed Dilution of Precision (SDOP) parameters newly introduced by SiRF. Direct measurements of GPS-Doppler speed errors are confronted with theoretical SDOP predictions. Comparison reveals that SDOP is a very useful and practical parameter for determining accuracy of GPS-Doppler speed measurement. It is demonstrated that 10-second average speed can be measured with accuracy better than 5 cm/s and confidence better than 99.9% using GT31 logger equipped with SiRF3 chipset.

Introduction

The GPS-Doppler method of speed measurement is very insensitive to atmospheric disturbances [1]. It is also the most direct and also the most convenient method of measuring speed, providing that bandwidth of the phase-lock-loop (PLL) receivers is well matched with the speed sampling rate to reduce/eliminate aliasing. [2]

Since the GPS-Doppler method of speed measurement is so direct and so convenient, it is necessary to develop methods of estimating its accuracy with high degree of confidence.

This article explores and tests the "Speed Dilution of Precision" (SDOP) parameters developed and kindly provided by SiRF (many thanks SiRF) and implemented by Locosys Technology Inc (many thanks Roger at Locosys) into firmware of their GT31 hand-held GPS data logger.

Verifying SDOP

SDOP is a new SiRF3 parameter determined on the basis of the Kalman Filter covariance computed during each 1-second cycle of the SiRF3 chipset. This theoretical "speed dilution of precision" estimate needs to be verified with Observable Reality before we can use it with confidence. An experimental SDOP verification method is needed.

When a GPS chipset is perfectly stationary in the geocentric frame of reference, its real speed with respect to the ground is exactly zero. Any speed with respect to ground that GPS chipset reports in such a stationary position is a direct measure of the GPS-Doppler speed error. This speed error can be considered a sum of all GPS-Doppler errors, including geometric, atmospheric, relativistic and position errors [1].

For this reason, GPS-Doppler speed data from a stationary chipset can be directly compared with theoretical SDOP predictions.

The graph in Fig 1 presents such a comparison for some 81,000 measurements of horizontal speed (SOG SiRF parameter). The resolution of both SOG (Speed Over Ground) and SDOP parameters is 1 cm/s, so that the entire graph consists of discrete points. It is important to note that among 81,000 actual speed error measurements not even a single speed error value was greater than the corresponding SDOP theoretical prediction. On average, the SDOP values are *more than 4 times larger* than observed speed errors. Results of this experiment suggest that SDOP provides a very conservative estimate of the observable speed error and hence it can be considered as an *upper bound* of actual GPS-Doppler speed errors with very high confidence.

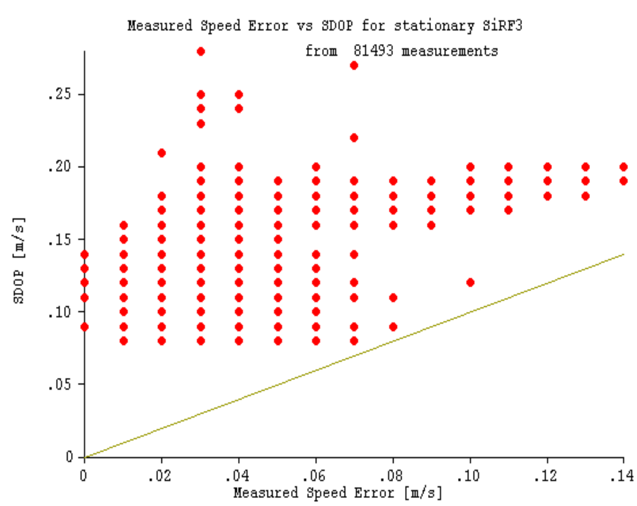


Fig 1. Comparison of SDOP with the actual observed (measured) speed error. Straight line is a line for which (speed error)=SDOP. All points are above this line, which means that for all 81,493 measurements the SDOP prediction was greater than the measured speed error. The average ratio of SDOP/(speed error) = 4.3

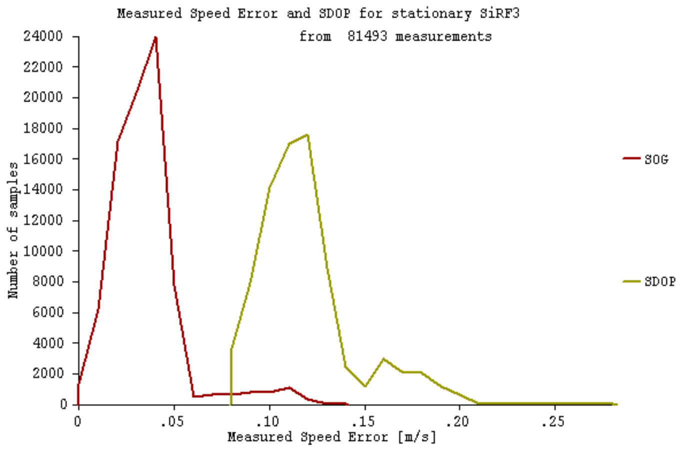


Fig 2. Comparison of measured horizontal speed error (speed over ground SOG) and SDOP parameter histograms for 81,493 observations.

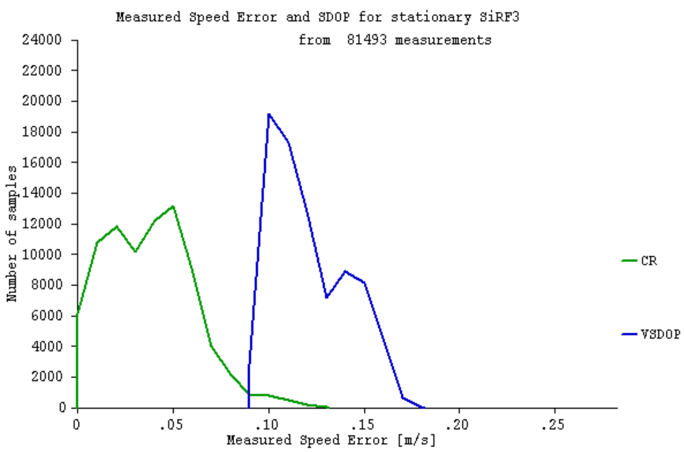


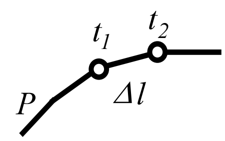
Fig 3. Comparison of measured vertical speed error (Climb Rate CR) and VSDOP parameter histograms for 81,493 observations.

During 2 days of testing two GT31 units were used at 4 different geostationary locations and experienced temperature variations between 15 and 45 degrees Celsius. SiRF3 chipsets experienced a variety of satellite configurations and atmospheric conditions, so that reported data should represent quite well the actual GPS-Doppler performance of SiRF3 chipsets and GT31 units that implement them.

It is important to point out that, from the point of view of the GPS chipset receiver, GPS-Doppler measurement of zero-speed is no different from measuring any other speed. This is because Earth's surface spins, so that a point that seems stationary with respect to the surface of the Earth actually moves West-to-East with speed in the order of 350m/s (~700 knots) in the geocentric frame of reference. Satellites move even faster. So, a geostationary test of SDOP and VSDOP (vertical speed dilution of precision) described in this article should well represent the SDOP and VSDOP performance when measuring speeds of vehicles and sailing craft on the surface of Earth with the GPS Doppler method.

Implementing SDOP to average speed measurement

Let's begin with recalling the universally accepted definitions of speed and the average speed. Consider object moving along path *P* and traveling the distance Δl during the time interval $\Delta t = t_2 - t_1$.



The speed *v* of the object is defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta l}{\Delta t} = \frac{dl}{dt} \qquad (1)$$

The average speed $\hat{v}$ over time *T* is defined as a time average of *v* as follows:

$$\hat{v} = \frac{\int_{t=0}^{T} v dt}{T} = \frac{\int_{0}^{L} dl}{T} = \frac{L}{T} \qquad (2)$$

Definition of the average speed determines unique relationship between the travelled distance *L* (measured along the path *P*) and the average speed $\hat{v}$

$$L = \hat{v} T \qquad (3)$$

It is important to point out that the average speed over the time *T* and the average speed over the distance *L* are identical.

In real measurements, measured GPS-Doppler speed v_D contains measurement error v_e. Hence, true speed is $v = v_D - v_e$. The true average speed over time T is then

$$\hat{v} = \frac{\int_{t=0}^{T} v_D dt}{T} - \frac{\int_{t=0}^{T} v_e dt}{T} = \hat{v}_D - \hat{v}_e \qquad (4)$$

The first term $\hat{v}_D$ represents the measured average Doppler speed; the second term $\hat{v}_e$ represents the measurement error.

For discrete measurements v_{kD} of speed v_D the first integral in equation (4) can be replaced by trapezoidal integration [5] as follows:

$$\hat{v}_D = \frac{\sum_{k=0}^{N} v_{kD}}{N} - \frac{v_{0D} + v_{ND}}{2N} \qquad (5)$$

where v_{0D} and v_{ND} are the first and the last samples in the integration interval T and N is the number of sampling intervals in T.

The exact discrete speed error values v_{ke} of v_e in equation (4) are actually unknown, but we found that SDOP values are excellent estimates for the *upper bound* of the magnitude of the speed error of each and every sample.

Hence, using SDOP values for each speed in the interval T we can compute the upper bound of the speed error

$$\hat{v}_{eU} = \frac{\sum_{k=0}^{N} SDOP_k}{N} - \frac{SDOP_0 + SDOP_N}{2N} \qquad (6)$$

where $SDOP_0$ and $SDOP_N$ are the SDOP values corresponding to the first and the last speed samples v_{0D} and v_{ND} in the integration interval T. Trapezoidal integration of SDOP ensures that SDOP values are given the same weighting as the corresponding speed samples v_{kD} in equation (5). Since each SDOP overestimates the actual speed error for each and every point, the actual error of the average speed $\hat{v}_e$ *must* be smaller than $\hat{v}_{eU}$.

100% confidence level?

Tolerances of the N-second average speeds that provide 100% confidence level can be obtained by seeking $\alpha > 0$ such that $\hat{v}_{eU} \geq \alpha \hat{v}_e$ is true for 100% of observed N-second sequences of direct geostationary measurements of speed error $\hat{v}_e$. Analysis of ~81000 such sequences from geostationary test data leads to the following result for Speed Over Ground (SOG):

$$\hat{v} = \hat{v}_D \pm \frac{\hat{v}_{eU}}{\alpha} \qquad (7)$$

Where $\alpha =$ 1.57851243, 1.58749998, 1.61843967 for 10s, 20s and 60s average speeds respectively.

Speed tolerances computed this way are larger than those computed using variances and statistical theorems, but seem to guarantee 100% confidence level in measured results. These tolerances have been found by exploring bounds of observable errors, and did not require assumptions that are required by variance-based statistical methods. Specifically, an assumption of statistical independence of variables (speed errors v_{ke} in our case) has not been required.

Errors v_{ke} in GPS-Doppler speed measurement can be considered sums of geometric, atmospheric, relativistic and position errors as well as satellite frequency-tracking errors [1]. Clearly, at least some of these GPS-Doppler speed error components are common for N consecutive speed errors v_{ke} so, strictly speaking, speed errors v_{ke} in such sequences may not be statistically independent.

99.9% confidence level

When 100% confidence level is not required, the Central Limit Theorem [4] can be used to compute a more practical value for the accuracy of the average speed, *providing* that measurement errors v_{ke} of each sample can be considered a series of independent random variables that share the same probability distribution D. Data presented earlier in this article demonstrates that both the expected value μ and the standard deviation σ of D exist and are finite. The probabilities of errors v_{ke} increasing and decreasing the measured values of speed can be assumed similar. Hence discrete random variable v_{ke} can be considered to have $\mu = 0$.

The standard error $\sigma_{\hat{v}}$ of the measured average speed $\hat{v}$ can be determined directly from the Central Limit Theorem [4] as follows:

$$\sigma_{\hat{v}} = \sigma_{\hat{v}_e} = \frac{\sigma}{\sqrt{N}} \qquad (8)$$

Our problem is that we do not know σ. We only know SDOP and its trapezoidal average $\hat{v}_{eU}$, which clearly overestimates the upper bound of the average speed error of $\hat{v}$. However, from the geostationary data presented earlier in this article it is quite clear, that $\hat{v}_{eU}$ must be at least several times larger than σ. Let's assume that $\hat{v}_{eU} = c\sigma$, where $c>1$ is a constant. Then, we can estimate the $c\sigma_{\hat{v}}$ as follows:

$$c\sigma_{\hat{v}} = c\sigma_{\hat{v}_e} = \frac{\hat{v}_{eU}}{\sqrt{N}} , \qquad (9)$$

because we can multiply both sides of equation (8) by a constant c and substitute $\hat{v}_{eU} = c\sigma$. So, the measured averaged speed can be determined as

$$\hat{v} = \hat{v}_D \pm \frac{\hat{v}_{eU}}{\sqrt{N}} \quad (10)$$

where $\hat{v}_D$ and $\hat{v}_{eU}$ are defined by equations (5) and (6). It is difficult to determine what the exact level of confidence of the measured speed is, because the actual distribution of speed errors v_{ke} during the measurement remains unknown.

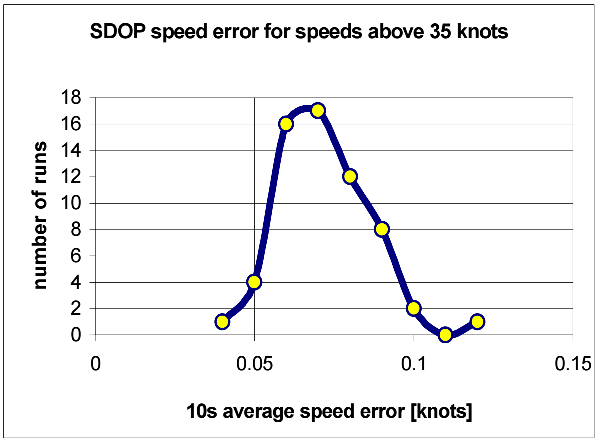


Fig. 4. SDOP-based speed error for 10s average speeds above 35 knots.(speed sailing data) determined using (10)

However, since magnitudes of SDOP predictions are on average more than 4 times greater than observable magnitudes of speed errors, standard deviation σ_{sdop} of the (signed) *sdop* distribution should be at least 3 times larger than σ: $\sigma_{sdop} > 3\sigma$, even if only the absolute values SDOP of *sdop* are known. For a probability density distribution with finite mean value the average value of the absolute values of N samples is larger than their standard error (simply because $\sum_N |x_i| \geq \sqrt{\sum_N x_i^2}$ and hence $\sum_N |x_i| \geq \sqrt{\sum_N \left(x_i - \frac{1}{N} \sum_N x_i \right)^2}$ for any $x_i \in \Re$), so $\hat{v}_{eU} \geq \sigma_{sdop}$. This means that $\hat{v}_{eU} > 3\sigma$ and that the corresponding confidence level associated with the speed error estimate (10) is better than 99.9%.

Example

Table 1 presents an example of a sailing speed measurement logged with GT31. Speeds in the table are in knots (nautical miles per hour).

The 10-second average speed over ground (the Doppler column) is 39.863 ± 0.064 knots and the corresponding average vertical speed (climb rate) is -0.021 ± 0.064 knots with 99.9% confidence. If 100% confidence level is required, the speed over ground should be considered to be 39.863 ± 0.127 knots from equation (7).

Track	Time	HDOP	Sats	Doppler	Vdoppler	SDOP	VSDOP
1282	15:16.0	0.8	9	39.149028	0.019438	0.194	0.194
1283	15:17.0	0.8	9	39.965443	0.077754	0.194	0.194
1284	15:18.0	0.8	9	40.742981	0.116631	0.194	0.194
1285	15:19.0	0.8	9	40.548596	-0.077754	0.194	0.194
1286	15:20.0	0.8	9	40.023758	-0.019438	0.194	0.194
1287	15:21.0	0.8	9	39.809935	0.019438	0.194	0.194
1288	15:22.0	0.8	9	39.634989	0.058315	0.194	0.194
1289	15:23.0	0.8	9	39.557235	0.058315	0.214	0.214
1290	15:24.0	0.8	9	39.634989	-0.019438	0.214	0.214
1291	15:25.0	0.8	9	39.596112	-0.058315	0.214	0.214
1292	15:26.0	0.8	9	39.091	0.097	0.214	0.214

Table 1. Typical 1-second samples of speed sailing data

Typical distribution of SDOP-based speed errors for 10s-average speeds above 35 knots is presented in Fig.4. Speed measurements were performed during 3 days of varying atmospheric conditions, including storms and rain. Two GT31 loggers mounted in the helmet experienced variety of satellite configurations. As can be seen from Fig 4, speed errors below 0.04 knots (2 cm/s) have not been observed and speed errors above 0.1 knots (5 cm/s) were quite rare (~1.6% of samples).

Speed Accuracy during Speed Sailing

Speed sailing is done on perfectly flat water so that the speed in the vertical direction is always very close to zero, which is well illustrated in the example presented in Table 1.

GPS-Doppler speed measurement using GT31 (SiRF3 chipset) is inherently 3D and hence SDOP is always intimately correlated with VSDOP. For this reason, in speed sailing on flat water, the vertical speed and VSDOP estimates can be used to scale the SDOP to determine the accuracy of each speed sample individually. Providing that speed over ground (SOG, reported as Doppler in the example above) is high enough and that the corresponding vertical speed v_{kV} is smaller than 0.1 m/s or ~0.2 knots (maximum observable speed during geo-stationary tests) for all vertical speed samples, we can estimate the speed error for each and every speed sample in a "speed run" as follows:

$$v_{ke} = \frac{v_{kV}}{VSDOP_k} SDOP_k \qquad (11)$$

The sign of the SDOP in relation to the sign of the speed error cannot be verified. For this reason we can only consider *absolute* values of v_{ke}. The average of absolute values of v_{ke} for the example above gives the 10-second average speed tolerance of

$$\frac{\sum_{k=0}^{N} |v_{ke}|}{N} = \pm 0.056 \text{ knots} \qquad (12)$$

It is important to note that this result assumes that each and every speed sample in the "speed run" had a maximum possible error $|v_{ke}|$ and that all errors have added up. The probability that all N errors would add up is 0.5^N. For N=10, this probability is $0.5^{10} = 0.000976$. So, providing that values of $|v_{ke}|$ are realistic, this probability guarantees the confidence level 99.9% of the result (12) for 10-second average speed.

During speed sailing while windsurfing (or kite surfing) the zero-vertical-speed can be perturbed. These perturbations may arise because GPS loggers are typically worn on the body of the sailor and the position of the body over the water during speed sailing may change due to wind gusts and the required balance.

Equation (12) shows that (in the method presented in this section) any non-zero value of physical vertical speed will be considered as a horizontal speed error and hence, when horizontal speed perturbations are present, the equation (12) will over-estimate the actual speed error.

The method presented in this section suggests, that horizontal speeds measured during Speed Sailing can be used to compute an estimate for the horizontal average speed errors, even when SDOP and VSDOP vales are not available (in historic data for example).

Conclusions

Speed measurement using GPS-Doppler turns out to be the most accurate GPS kinematic measurement of the GT31 logger equipped with the SiRF3 chipset. 10-second average speed accuracy better than 5 cm/s with the confidence level better than 99.9% has been observed.

Benefits of using SDOP and VSDOP for determining errors of GPS-Doppler speed measurements include

1. SDOP parameter reflects the influence of atmospheric phenomena and satellite constellation geometry at the time of each and every GPS-Doppler speed measurement.
2. GPS Doppler speed accuracy can be computed for each speed measurement individually. This is important when speeds are measured in varying atmospheric and other conditions
3. The method of evaluating speed measuring errors on the basis of SDOP parameters is self-adjusting to measurement conditions
4. Speed values displayed by GPS unit in the real time can be corrected for the speed error. Such a correction, when used for vehicle speed monitoring, can help drivers to make sure that they do not exceed speed limits.
5. Logged GPS-Doppler speeds corrected by SDOP can serve as proof of speed.
6. In sports involving speed-based rankings, correcting for speed errors can guarantee that claimed speeds have indeed been achieved with high certainty.
7. During extreme atmospheric conditions (storms) one GT31 logger could be used in a geo-stationary position to provide reassurance about SDOP and VSDOP statistics.

The article presented speed accuracy analysis for the horizontal speed (speed over ground SOG), and vertical speed separately but can easily be extended to 3D speed accuracy analysis.

Acknowledgments

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Many thanks to Mal Wright, the author of RealSpeed software [8] for his help in analysing speed sailing data and testing algorithms presented in this article.

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Appendix 1. SiRF3 navigation parameters

The list of SiRF3 chipset navigation parameters used for tests presented in this article has been as follows:

1. AltMode: never use
2. AltSource: last KF alt
3. Altitude: 0
4. DegradedMode: Disabled
5. DegradedTimeout: 30 s
6. DRTimeout: 0 s
7. TrkSmoothMode: disabled
8. StaticNav: disabled
9. 3SV LSQ: enabled
10. DOPMaskMode: auto
11. ElevMask:10.0 deg
12. PwrMask: 12 dBHz
13. DGPSSrc: None
14. DGPSMode: auto
15. DGPSTimeout: 0 s
16. Continuous power enabled
17. User tasks enabled, period = 100
18. MaxAcqTime = 120000 ms
19. MaxOffTime = 30000 ms

It was found that the reproducibility and repeatability of SiRF3 Doppler speed measurement is best when parameters 1,3,4,6,7,8,13 are all set as listed above. Setting any of the parameters 1,4,7,8,13 differently than listed above significantly degrades SiRF3 Doppler speed measurement accuracy and reproducibility.

Appendix 2 : modifications to SBP and SBN data formats

SBN: Two bytes appended to message_0x29
UINT8 SDOP;   (in cm/sec)
UINT8 VSDOP; (in cm/sec)

The total length of SBN record written to SD card is now 97 bytes, with the last 2 bytes representing SDOP and VSDOP.

SBP: The last two bytes in 32-byte record are redefined as follows

UINT8 SDOP;   (in cm/sec)
UINT8 VSDOP; (in cm/sec)

0xff will be recorded if the return value is over 255 or SDOP/VSDOP values cannot be computed.

Appendix 3. Course Over Ground (COG) error

The Course Over Ground (COG) error COGDOP can be estimated on the basis of SDOP as follows:

$$COGDOP=\arctan(SDOP/speed) \qquad (3.1)$$

Where "speed" is the Speed over Ground (SOG) ± speed error. From equation (3.1) it is clear that Course Over Ground (COG) error decreases when speed increases and than for small speeds the error rapidly grows.

An attempt to find "direction" or "bias" for speed error on the basis of geo-stationary measurements of SOG and COG cannot be considered useful, because COGDOP errors in such a situation are very large.

Since directional errors COGDOP for near-zero speeds can be very large, only magnitudes of speed errors observed during geo-stationary tests are useful and relevant to speed error calculation.